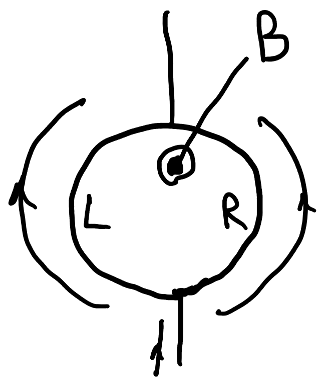
Aharonov-Bohm effect. Effect of magnetic field on WL.

Consider a setup consisting of a ring, made of 1D wires, threaded by magnetic flux



Quasiparticles propagate along one of two arms and then interfere

$$\varphi_L = \int_L (\vec{p} - \frac{e}{c}\vec{A})\, d\vec{r} / \hbar$$

$$\varphi_R = \int_R (\vec{p} - \frac{e}{c}\vec{A})\, d\vec{r} / \hbar$$

Phase difference $\Delta\varphi = \varphi_L - \varphi_r =$

$$= \left[\frac{e}{c}\int_R \vec{A}\, d\vec{r} - \frac{e}{c}\int_L \vec{A}\, d\vec{r}\right] / \hbar = \frac{e}{c\hbar}\oint \vec{A}\, d\vec{r} =$$

$$= \frac{e}{c\hbar}\Phi = 2\pi\frac{\Phi}{\Phi_0}$$

where we have introduced $\Phi_0 = \frac{2\pi c\hbar}{e} \equiv \frac{ch}{e}$ - <u>magnetic flux quantum</u>

Now the total amplitude

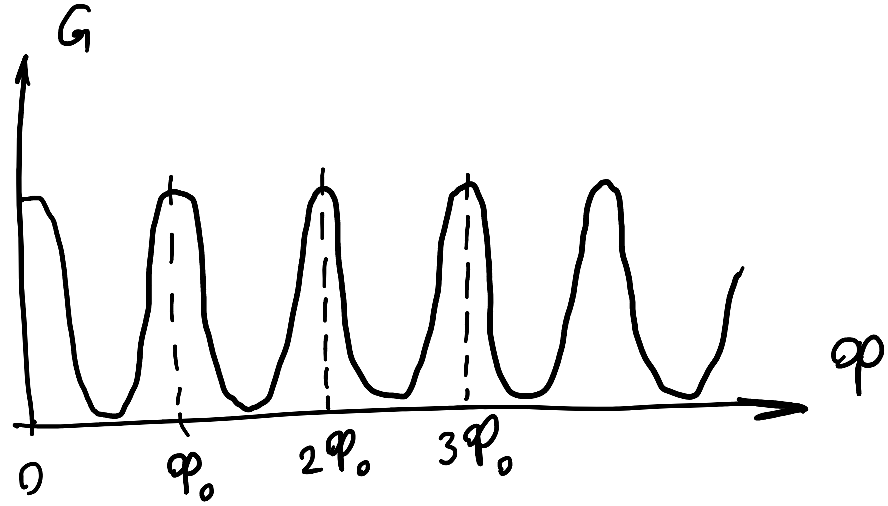
$$\ldots e^{-i\pi\frac{\Phi}{\Phi_0}} + e^{i\pi\frac{\Phi}{\Phi_0}} = \cos(\pi\frac{\Phi}{\Phi_0})$$

Now the wave

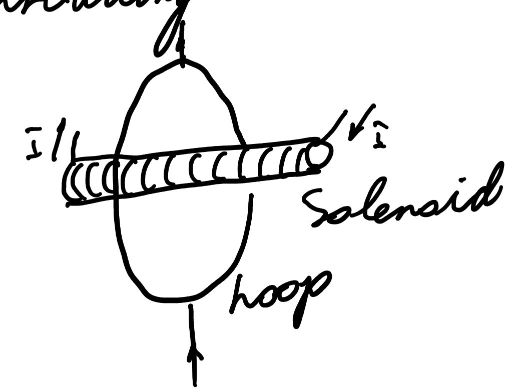
$$\psi = \psi_L + \psi_R \sim e^{-i\pi\frac{\varphi}{\varphi_0}} + e^{i\pi\frac{\varphi}{\varphi_0}} = \cos\left(\pi\frac{\varphi}{\varphi_0}\right)$$

$$|\psi|^2 \sim \cos^2\left(\pi\frac{\varphi}{\varphi_0}\right) \sim 1 + \cos\left(2\pi\frac{\varphi}{\varphi_0}\right)$$

Conductance



Note: Aharonov-Bohm oscillations exist even when the magnetic field does not reach the wires; it may be confined to the solenoid which is threading the wire loop



$$\oint \vec{A}\, d\vec{r} = \varphi$$

Osakabe experiment: (1986)

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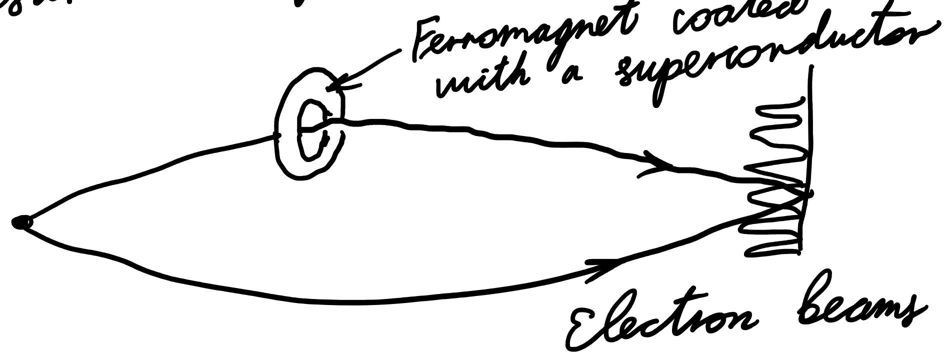
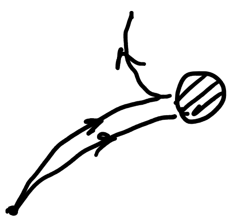


Effect of magnetic field on WL

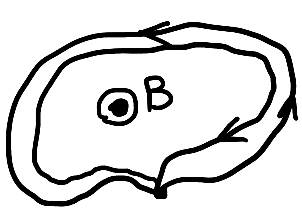
Effect of magnetic field on transport:
magnetic field makes trajectories curve

$$\omega_c = \frac{eB}{mc} \quad \text{- cyclotron frequency}$$



If $\omega_c \tau \ll 1$, then there is another much stronger effect: effect on WL

WL comes from the interference of pairs of trajectories propagating in opposite directions



$$|A_1 + A_2|^2 =$$

$$= |A_1|^2 + |A_2|^2 + \underbrace{A_1^* A_2 + A_2^* A_1}_{2\cos\left(\pi \frac{\varphi}{\varphi_0}\right)}$$

$$\left\langle \cos\left(\pi \frac{\varphi}{\varphi_0}\right) \right\rangle = 0$$

$$\left\langle \cos\left(\pi \frac{\varphi}{\varphi_0}\right)\right\rangle_{\text{all trajectories}} = 0$$

→ Magnetic field destroys WL

Focus on 2D

The characteristic "interference area" is

$$S_\varphi = D\tau_\varphi$$

$$\to \boxed{B_\varphi = \frac{c\hbar}{e}\frac{1}{D\tau_\varphi}}$$

If we write $D \sim \frac{\varepsilon_F \tau}{m}$ then

$$\underbrace{\frac{eB_\varphi}{mc}}_{=\,\omega_c}\tau\frac{\tau_\varphi\varepsilon_F}{\hbar}\approx 1 \to \omega_c\tau \underbrace{\frac{\varepsilon_F\tau_\varphi}{\hbar}}_{\gg 1 \text{ in a good metal}} \approx 1$$

$$\to \omega_c\tau \ll 1$$